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CLAIMS

What is claimed is:

1. (Cancelled)
2. A method for computing the conformal structure of a surface, the method comprising:
  - receiving a first mesh representation M of said surface;
  - conformally mapping said mesh representation to a canonical parameter domain forming a first mapped surfaces; and
  - computing a conformal parameterization of said first mapped surface.
3. The method of claim 1 wherein the step of computing said conformal parameterization includes:
  - computing a homology basis  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  of said mesh representation, wherein computing said homology basis includes the steps of:
    - computing a set of boundary matrices  $\partial_1, \partial_2$  for said mesh representation, respectively;
    - forming a matrix D, wherein  $D = \partial_2 \partial_2^T + \partial_1^T \partial_1$ ;
    - computing the Smith normal for said matrix D; and
    - computing the eigenvectors of D corresponding to zero eigenvalues that are equal to  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$ , and wherein  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  are a homology basis of said mesh representation.
4. The method of claim 2 wherein the step of computing the conformal parameterization includes the step of computing a set of harmonic 1-form basis, wherein the step of computing said set of harmonic 1-form basis includes the steps of:

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calculating the value of  $c_i^j = -\gamma_i \bullet \gamma_j$ ,  $i, j = 1, 2, \dots, 2g$  wherein  $\gamma_i, \gamma_j$  is the homology basis of the mesh representation; and

solving the linear system  $\delta\omega_i = 0, \Delta\omega_i = 0, \langle\omega_i, \gamma_j\rangle$   
 5  $= -\gamma_i \bullet \gamma_j$  for  $\omega_i$ , wherein  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$  is the desired harmonic 1-form basis of said mesh representation.

5. The method of claim 3 wherein the step of computing the conformal parameterization further includes the steps of:

10 computing the fundamental domain  $D_M$  of the mesh representation, wherein the step of computing the fundamental domain  $D_M$  of the mesh representation includes the steps of:

selecting an arbitrary face,  $f_0 \in M$ ;

setting  $D_M$  to  $f_0$ ;

15 setting  $\partial D_M = \partial f_0$ ;

placing all neighboring faces of  $f_0$  that share an edge with  $f_0$  in a queue,  $Q$ ;

while  $Q$  is not empty:

reading the first face in  $Q$ ;

20 setting  $\partial f = e_0 + e_1 + e_2$ ;

setting  $D_M = D_M \cup f$ ;

finding the first  $e_i \in \partial f$  such that  $-e_i \in \partial D_M$ ;

replacing  $-e_i$  in  $\partial D_M$  with  $\{e_{(i+1)}, e_{(i+2)}\}$ ;

putting in  $Q$  all the neighboring faces that share an  
 25 edge with  $f$  in the mesh representation and that are not in  $D_M$  or  $Q$ ; and

removing all adjacent oriented edges in  $\partial D_M$  that are opposite each other in sign.

6. The method of claim 4 wherein the step of calculating said conformal parameterization of said first mapped surface further includes the steps of:

recording a first path  $\delta$  from root vertex  $v_0$  to  $u$  by  
 5 traversing all the vertices  $u \in D_M$ ;  
 computing the integration of  $\phi(u) = \langle \omega, \gamma_u \rangle$ ; and  
 providing as an output  $\phi(u)$  as the conformal coordinates of  
 $u$ .

10 7. The method of claim 5, wherein the step of calculating said conformal parameterization of said first mapped surface further includes the step of calculating the cohomology basis, wherein the step of calculating the cohomology basis includes the steps of:

15 setting  $\omega_i(e_i)=1$  and  $\omega_i(e_j)=0$  for any edge  $e \in T$ ;  
 ordering  $D_M$  such that  $D_M = \{f_1, f_2, \dots, f_n\}$ ;  
 reversing the order of  $D_M$  to  $\{f_n, f_{n-1}, \dots, f_1\}$ ;  
 while  $D_M$  is not empty:  
 retrieving the first face  $f$  of  $D_M$ ;  
 20 removing  $f$  from  $D_M$ ,  $\partial f = e_0 + e_1 + e_2$ ;  
 divide  $\{e_k\}$  into two sets,  $\Gamma = \{e \in \partial f \mid -e \in \partial D_M\}$ ,  
 $\Pi = \{e \in \partial f \mid -e \notin \partial D_M\}$ ;

choosing the value of  $\omega_i(e_k), e_k \in \Pi$  arbitrarily, such that  
 $\sum_{e \in \Pi} \omega_i(e) = -\sum_{e \in \Gamma} \omega_i(e)$ , and if  $\Pi$  is empty, then the right hand side  
 25 is equal to zero;

updating the boundary of  $D_M$ , let  $\partial D_M = \partial D_M + \partial f$ ; and  
 computing the dual of the homology basis,  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$ , by  
 the linear transform  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$  such that  $\langle \gamma_i, \omega_j \rangle = -\gamma_i \cdot \gamma_j$ .

8. The method of claim 1, wherein the step of conformally mapping and calculating said conformal parameterization of said mesh representation includes the step of calculating the holomorphic 1-form, wherein the step of calculating the  
5 holomorphic 1-form includes the steps of:

computing the doubling of  $M$ ,  $\bar{M}$ ;  
computing the harmonic 1-form basis of  $\bar{M}$   $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$ ;  
assigning  $\tau_i = \frac{1}{2}(\omega + \bar{\omega})$  and removing redundant ones;  
computing conjugate harmonic 1-forms of  $\tau_i$  denoted as  $\tau_i^*$ ; and  
10 outputting the holomorphic 1-form basis  
 $\{\tau_1 + \sqrt{-1}\tau_1^*, \tau_2 + \sqrt{-1}\tau_2^*, \dots, \tau_k + \sqrt{-1}\tau_k^*\}$ .

9. The method of claim 7 wherein the step of doubling  $M$  to form  $\bar{M}$  includes the steps of:

15 copying  $M$ , denoted as  $-M$ ;  
reversing the orientation of  $-M$ ;  
finding for each boundary vertex  $u \in \partial M$ , a unique corresponding boundary vertex  $-u \in \partial -M$ ;  
finding for any edge on  $e \in \partial M$  a unique boundary edge  $-e \in \partial$   
20  $-M$ ; and  
gluing  $M$  and  $-M$  such that the corresponding vertices and edges are identical, wherein the resulting mesh is the doubling  $\bar{M}$ .

25 10. The method of claim 7 wherein the step of calculating said conformal parameterization of said mesh representation,  $M$ , further includes the step of calculating a conformal structure, wherein the step of calculating the conformal structure includes the steps of:

30 a) providing the holomorphic 1-form basis  $\{\tau_i + \sqrt{-1}\tau_i^*\}$ ;

- b) computing a partition  $\{U_i\}$ , such that  $M \subset U_i$ ,  $U_i$  being simply connected;
- c) for each  $U_i$ , selecting a holomorphic 1-form base  $\omega_j + \sqrt{-1}\omega_j^*$ ;
- 5 d) integrating the holomorphic 1-form on  $U_i$ , denoting the mapping as  $z_i$ , and in the event that there are zero points, subdivide  $U_i$  and repeat the integration; and
- e) outputting the conformal structure  $\{(U_i, z_i)\}$ .
- 10 11. The method of claim 4 wherein the step of calculating said conformal parameterization of said mesh representation further includes the step of calculating the period matrices, wherein the step of calculating the period matrices includes the steps of:
- calculating the elements of a matrix C as  $c_{ij} = \langle \gamma_i, \omega_j \rangle$ ;
- 15 calculating the elements of a matrix S as  $s_{ij} = \langle \gamma_i, \omega_j^* \rangle$ ; and
- calculating the matrix R as the solution of  $CR = S$  where R satisfies  $R^2 = -I$ , where I is the identity matrix.
- 20 12. A method for classifying a surface, the method comprising the steps of:
- receiving a mesh representation of the surface;
- computing a period matrix R corresponding to said mesh representation of the surface; and
- 25 storing said period matrix R.
13. The method of claim 11 wherein the step of computing said period matrix R includes the steps of:
- computing a homology basis  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  of said mesh
- 30 representation, wherein computing said homology basis includes the steps of:

computing a boundary matrix  $\partial_1, \partial_2$  for said mesh representation;

forming a matrix  $D$ , wherein  $D = \partial_2 \partial_2^T + \partial_1^T \partial_1$ ;

computing the Smith normal for said matrix  $D$ ; and

5        computing the eigenvectors of  $D$  corresponding to zero eigenvalues that are equal to  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  wherein  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  are a homology basis of said mesh representation.

14. The method of claim 12 wherein the step of calculating said  
10    period matrix  $R$  further includes the step of computing a set of harmonic 1-form basis, wherein the step of computing said set of harmonic 1-form basis includes the steps of:

      calculating the value of  $c_i^j = -\gamma_i \bullet \gamma_j, i, j = 1, 2, \dots, 2g$ , wherein  $\gamma_i, \gamma_j$  are the homology basis of the mesh  
15    representation; and

      solving the linear system  $\delta \omega_i = 0, \Delta \omega_i = 0, \langle \omega_i, \gamma_j \rangle = -\gamma_i \bullet \gamma_j$  for  $\omega_i$ , wherein  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$  is the desired harmonic 1-form basis of the mesh representation.

20    15. The method of claim 13 wherein the step of computing said period matrix  $R$  further includes:

      calculating the elements of a matrix  $C$  as  $c_{ij} = \langle \gamma_i, \omega_j \rangle$ ;

      calculating the elements of a matrix  $S$  as  $s_{ij} = \langle \gamma_i, \omega_j^* \rangle$ ; and

25        calculating the period matrix  $R$  as the solution of  $CR = S$  where  $R$  satisfies  $R^2 = -I$ , where  $I$  is the identity matrix.

16. A method for matching first and second surfaces, the method comprising:

30        receiving first and second mesh representations of said first and second surfaces, respectively;

conformally mapping both of said first and second mesh representations to a first canonical parameter domain, thereby forming first and second mapped surfaces;

5 computing first and second conformal parameterizations of said first and second mapped surfaces, respectively;

computing first and second level sets of Gaussian curvature and mean curvature for said first and second mapped surfaces, respectively, wherein said first and second level sets of Gaussian curvature and mean curvature are a function of said  
10 first and second conformal parameterizations, respectively; and

comparing said first and second level sets of Gaussian curvature and mean curvature and in the event that the comparison exceeds a predetermined threshold, declare a match between said first and second surfaces, otherwise declare a mismatch between  
15 said first and second surfaces.

17. The method of claim 15 wherein the step of computing said first and second conformal parameterizations includes the step of:

20 computing a homology basis  $\{\gamma_{11}, \gamma_{12}, \dots, \gamma_{12g}\}$  and  $\{\gamma_{21}, \gamma_{22}, \dots, \gamma_{22g}\}$  of said first and second mesh representations, respectively, wherein computing said homology basis includes the steps of:

computing first and second sets of first and second  
25 boundary matrices  $\partial_{11}, \partial_{12}$  and  $\partial_{21}, \partial_{22}$  for said first and second mesh representations, respectively;

forming first and second matrices  $D_1$  and  $D_2$ , respectively, wherein  $D_1 = \partial_{12}\partial_{12}^T + \partial_{11}^T\partial_{11}$  and  $D_2 = \partial_{22}\partial_{22}^T + \partial_{21}^T\partial_{21}$ ;

30 computing the Smith normal for said matrices  $D_1$  and  $D_2$ ; and

computing the eigenvectors of  $D_1$  and  $D_2$  corresponding to zero eigenvalues that are equal to  $\{\gamma_{11}, \gamma_{12}, \dots, \gamma_{12g}\}$  and  $\{\gamma_{21}, \gamma_{22}, \dots, \gamma_{22g}\}$  wherein  $\{\gamma_{x1}, \gamma_{x2}, \dots, \gamma_{x2g}\}$  are a homology basis of said first and second mesh representations, respectively.

18. The method of claim 16 wherein the step of computing said first and second conformal parameterizations further includes the step of computing a first and second set of harmonic 1-form basis, wherein the step of computing said first and second set of harmonic 1-form basis includes the steps of:

calculating the value of  $c_{1i}^j = -\gamma_{1i} \bullet \gamma_{1j}$ ,  $i, j = 1, 2, \dots, 2g$  and  $c_{2i}^j = -\gamma_{2i} \bullet \gamma_{2j}$ ,  $i, j = 1, 2, \dots, 2g$  wherein  $\gamma_{1i}$ ,  $\gamma_{1j}$  and  $\gamma_{2i}$ ,  $\gamma_{2j}$  are the homology basis of the first and second mesh representations, respectively;

solving the linear system  $\delta\omega_{1i} = 0$ ,  $\Delta\omega_{1i} = 0$ ,  $\langle\omega_{1i}, \gamma_{1j}\rangle = -\gamma_{1i} \bullet \gamma_{1j}$  for  $\omega_{1i}$ , wherein  $\{\omega_{11}, \omega_{12}, \dots, \omega_{12g}\}$  is the desired harmonic 1-form basis of the first mesh representation; and

solving the linear system  $\delta\omega_{2i} = 0$ ,  $\Delta\omega_{2i} = 0$ ,  $\langle\omega_{2i}, \gamma_{2j}\rangle = -\gamma_{2i} \bullet \gamma_{2j}$  for  $\omega_{2i}$ , wherein  $\{\omega_{21}, \omega_{22}, \dots, \omega_{22g}\}$  is the desired harmonic 1-form basis of the second mesh representation.

19. The method of claim 17 wherein the step of computing said first and second conformal parameterizations further includes the step of computing the fundamental domain  $D_{1M}$  and  $D_{2M}$  of the first and second mesh representations respectively, wherein the step of computing the fundamental domain  $D_{1M}$  and  $D_{2M}$  of the first and second mesh representations respectively includes the steps of:

selecting an arbitrary face,  $f_{10} \in M_1$  and  $f_{20} \in M_2$ , and set  $D_{1M}$  and  $D_{2M}$  to  $f_{10}$  and  $f_{20}$  respectively;

setting  $\partial D_{1m} = \partial f_{10}$  and  $\partial D_{2m} = \partial f_{20}$ ;



placing all neighboring faces of  $f_{10}$  and  $f_{20}$  that share an edge with  $f_{10}$  and  $f_{20}$  respectively in a queue,  $Q_1$  and  $Q_2$ , respectively;

while  $Q_1$  and  $Q_2$  are not empty:

5           reading the first face in  $Q_1$  and  $Q_2$ ;  
          setting  $\partial f_1 = e_{10} + e_{11} + e_{12}$ ;  
          setting  $D_{1M} = D_{1M} \cup f_1$ ;  
          finding the first  $e_{1i} \in \partial f_1$  such that  $-e_{1i} \in \partial D_{1M}$ ;  
          replacing  $-e_{1i}$  and  $-e_{2i}$  in  $\partial D_{1M}$  and  $\partial D_{2M}$ , respectively,  
10   with  $\{e_{1(i+1)}, e_{1(i+2)}\}$  and  $\{e_{2(i+1)}, e_{2(i+2)}\}$ , respectively;  
          putting all the neighboring faces that share an edge with  $f_1$  and  $f_2$  in the first and second mesh representations, respectively, and that are not in  $D_{1M}$  and  $Q_1$  and  $D_{2M}$  and  $Q_2$ , respectively, into  $Q_1$  and  $Q_2$ , respectively; and  
15       removing all adjacent oriented edges in  $\partial D_{1M}$  and  $\partial D_{2M}$  that are opposite each other in sign.

20. The method of claim 15 wherein said first and second surfaces are human faces.

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21. The method of claim 15 wherein said canonical parameter domain is a disk.

22. A method for recognizing a surface represented by a mesh  
25   representation,  $M$ , the method comprising the steps of:

          receiving the mesh representation of the surface;  
          removing at least one feature point from the mesh representation;

          doubling the mesh representation,  $M$ ;

30       computing the period matrix,  $R$ , for the mesh representation,  $M$ , having at least one feature point removed;

comparing the period matrix,  $R$ , corresponding to the mesh representation with at least one standard period matrix corresponding to a standard surface;

in the event that the period matrix,  $R$ , corresponding to the  
5 mesh representation and the at least one standard period matrix corresponding to a standard surface are within a predetermined factor of one another, providing indicia that the surface represented by the mesh representation is recognized as being substantially similar to the at least one standard surface.

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23. The method of claim 21, wherein the step of doubling the mesh representation,  $M$ , includes the steps of:

copying  $M$ , denoted as  $-M$ ;

reversing the orientation of  $-M$ ;

15 finding for each boundary vertex  $u \in \partial M$  a unique corresponding boundary vertex  $-u \in \partial -M$ ;

finding for any edge on  $e \in \partial M$  a unique boundary edge

$-e \in \partial -M$ ; and

gluing  $M$  and  $-M$  such that the corresponding vertices and  
20 edges are identical, wherein the resulting mesh is the doubling  $\overline{M}$ .

24. The method of claim 21, wherein the step of computing the period matrix  $R$  includes the steps of:

25 computing a homology basis  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  of said mesh representations, wherein computing said homology basis includes the steps of:

computing a set of boundary matrices  $\partial_1, \partial_2$  for said mesh representations;

30 forming a matrix  $D$ , wherein  $D = \partial_2 \partial_2^T + \partial_1^T \partial_1$ ;

computing the Smith normal for said matrix  $D$ ; and

computing the eigenvectors of  $D$  corresponding to zero eigenvalues that are equal to  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  wherein  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  are a homology basis of said mesh representation.

- 5 25. The method of claim 23 wherein the step of calculating said period matrix  $R$  further includes the step of computing a set of harmonic 1-form basis, wherein the step of computing said set of harmonic 1-form basis includes the steps of:

calculating the value of  $c_i^j = -\gamma_i \bullet \gamma_j$ ,  $i, j = 1, 2, \dots, 2g$ , wherein  $\gamma_i, \gamma_j$  are the homology basis of the mesh representation; and

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solving the linear system  $\delta\omega_i = 0$ ,  $\Delta\omega_i = 0$ ,  $\langle\omega_i, \gamma_j\rangle = -\gamma_i \bullet \gamma_j$  for  $\omega_i$ , wherein  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$  is the desired harmonic 1-form basis of the mesh representation.

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26. The method of claim 24 wherein the step of computing said period matrix  $R$  further includes:

calculating the elements of a matrix  $C$  as  $c_{ij} = \langle\gamma_i, \omega_j\rangle$ ;  
calculating the elements of a matrix  $S$  as  $s_{ij} = \langle\gamma_i, \omega_j^*\rangle$ ; and

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calculating the matrix  $R$  as the solution of  $CR = S$  where  $R$  satisfies  $R^2 = -I$ , where  $I$  is the identity matrix.

27. A method for recognizing a surface represented by a mesh representation,  $M$ , the method comprising the steps of:
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receiving the mesh representation of the surface;  
removing substantially all feature points from the mesh representation;

selecting an arbitrary point on the surface of the mesh representation,  $M$ ;

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selecting an arbitrary orbit along the surface of the mesh  
 representation,  $M$ , for the arbitrary point to follow;  
 moving the arbitrary point along the arbitrary orbit;  
 removing the arbitrary point from the mesh representation,  
 5  $M$ , at at least one point along the arbitrary orbit;  
 doubling the mesh representation,  $M$ , having all feature  
 points removed and the arbitrary point removed;  
 computing at least one period matrix,  $R$ , for the mesh  
 representation having the arbitrary point removed;  
 10 comparing the at least one period matrix,  $R$ , corresponding  
 to the mesh representation with at least one standard period  
 matrix corresponding to a standard surface; and  
 in the event that the period matrix,  $R$ , corresponding to the  
 mesh representation and the at least one standard period matrix  
 15 corresponding to a standard surface are within a predetermined  
 factor of one another, providing indicia that the surface  
 represented by the mesh representation is recognized as being  
 substantially similar to the at least one standard surface.

20 28. The method of claim 26, wherein the step of doubling the  
 mesh representation,  $M$ , includes the steps of:  
 copying  $M$ , denoted as  $-M$ ;  
 reversing the orientation of  $-M$ ;  
 finding for each boundary vertex  $u \in \partial M$ , a unique  
 25 corresponding boundary vertex  $-u \in \partial -M$ ;  
 finding for any edge on  $e \in \partial M$  a unique boundary edge  
 $-e \in \partial -M$ ; and  
 gluing  $M$  and  $-M$  such that the corresponding vertices and  
 edges are identical, wherein the resulting mesh is the doubling  
 30  $\overline{M}$ .

29. The method of claim 26, wherein the step of computing the period matrix R includes the steps of:

computing a homology basis  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  of said mesh representation, wherein computing said homology basis includes  
5 the steps of:

computing a boundary matrix  $\partial_1, \partial_2$  for said mesh representation;

forming a matrix D, wherein  $D = \partial_2 \partial_2^T + \partial_1^T \partial_1$ ;

computing the Smith normal for said matrix D; and

10 computing the eigenvectors of D corresponding to zero eigenvalues that are equal to  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  wherein  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  are a homology basis of said mesh representation.

30. The method of claim 26 wherein the step of calculating said  
15 period matrix R further includes the step of computing a set of harmonic 1-form basis, wherein the step of computing said set of harmonic 1-form basis includes the steps of:

calculating the value of  $c_i^j = -\gamma_i \cdot \gamma_j, i, j = 1, 2, \dots, 2g$ , wherein  $\gamma_i, \gamma_j$  are the homology basis of the mesh  
20 representation; and

solving the linear system  $\delta \omega_i = 0, \Delta \omega_i = 0, \langle \omega_i, \gamma_j \rangle = -\gamma_i \cdot \gamma_j$  for  $\omega_i$ , wherein  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$  is the desired harmonic 1-form basis of the mesh representation.

25 31. The method of claim 29 wherein the step of computing said period matrix R further includes:

calculating the elements of a matrix C as  $c_{ij} = \langle \gamma_i, \omega_j \rangle$ ;

calculating the elements of a matrix S as  $s_{ij} = \langle \gamma_i, \omega_j^* \rangle$ ; and

30 calculating the period matrix R as the solution of  $CR = S$  where R satisfies  $R^Z = -I$ , where I is the identity matrix.

32. A method for compressing a mesh representation of a surface, the method comprising the steps of:

conformally mapping the mesh representation to a canonical  
5 shape;

representing the surface position vector on the canonical shape as a vector valued function;

finding the eigen functions of the Laplacian given by  
$$\Delta \omega(u) = \sum_{[u,v] \in K_1} w_{[u,v]} \omega([u,v]);$$

10 decomposing the vector valued function using the eigen functions of the Laplacian;

removing the high frequency components of the decomposed vector valued function; and

storing the low frequency components of the decomposed  
15 vector valued function as a compressed image of the mesh representation, M, of the surface.

33. The method of claim 31, wherein the surface is a genus-zero surface and the canonical shape is a sphere and wherein the step  
20 of conformally mapping the genus-zero surface to the sphere includes the following steps:

- a) computing the Gauss map, mapping M to  $S^2$ ;
- b) computing the Laplacian at each vertex u of M,  $\Delta \phi(u)$ ;
- c) projecting  $\Delta \phi(u)$  to the tangent space of  $\phi(u) \in S^2$ ;
- 25 d) updating  $\phi(u)$  along the negative projected  $\Delta \phi(u)$ ;
- e) computing the center of mass of  $X\phi(u)$ ,  $mc(\phi)$ ;
- f) shifting the center of mass to the center of  $S^2$ ;
- g) renormalizing  $\phi(u)$  to be on  $S^2$ ; and
- h) repeating steps b)-g) for all vertices, until the  
30 projected Laplacian equals zero.

34. The method of claim 32, wherein the Laplacian is  
$$\Delta\omega(u) = \sum_{[u,v] \in K_1} w_{[u,v]} \omega([u,v]).$$

35. A method for compressing a mesh representation, M, of a  
5 surface, the method comprising:

receiving the mesh representation;

conformally mapping mesh representation to a canonical  
parameter domain forming a mapped surface;

10 computing a conformal parameterization of said mapped  
surface;

computing a level set of Gaussian curvature and mean  
curvature for said mapped surface, wherein said level set of  
Gaussian curvature and mean curvature is a function of said  
conformal parameterization; and

15 storing said conformal parameterization of said mesh  
representation and said level set corresponding to said mesh  
representation.

36. The method of claim 34 wherein the step of computing said  
20 conformal parameterization includes the step of:

computing a homology basis  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  of said mesh  
representation, wherein computing said homology basis includes  
the steps of:

25 computing a set of first and second boundary matrices  
 $\partial_1, \partial_2$  for said mesh representation;

forming a matrix D, wherein  $D = \partial_2 \partial_2^T + \partial_1^T \partial_1$ ;

computing the Smith normal for said matrix D; and

30 computing the eigenvectors of D corresponding to zero  
eigenvalues that are equal to  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  wherein  $\{\gamma_1, \gamma_2, \dots, \gamma_{2g}\}$  is a homology basis of said mesh representation.

37. The method of claim 35 wherein the step of computing said conformal parameterization further includes the step of computing a harmonic 1-form basis, wherein the step of computing said harmonic 1-form basis includes the steps of:

5        calculating the value of  $c_i^j = -\gamma_i \bullet \gamma_j$ ,  $i, j = 1, 2, \dots, 2g$  wherein  $\gamma_i, \gamma_j$  is the homology basis of the mesh representation respectively; and

      solving the linear system  $\delta\omega_i = 0, \Delta\omega_i = 0, \langle\omega_i, \gamma_j\rangle = -\gamma_i \bullet \gamma_j$  for  $\omega_i$ , wherein  $\{\omega_1, \omega_2, \dots, \omega_{2g}\}$  is the desired harmonic 1-form  
10 basis of the mesh representation.

38. The method of claim 36 wherein the step of computing said conformal parameterization further includes the step of computing the fundamental domain  $D_M$  of the mesh representation, wherein the  
15 step of computing the fundamental domain  $D_M$  of the mesh representation includes the steps of:

      selecting an arbitrary face,  $f_0 \in M$ , and set  $D_M$  to  $f_0$ ;

      setting  $\partial D_M = \partial f_0$ ;

      placing all neighboring faces of  $f_0$  that share an edge with  
20  $f_0$  in a queue,  $Q$ ;

      while  $Q$  is not empty:

          reading the first face in  $Q$ ;

          setting  $\partial f = e_0 + e_1 + e_2$ ;

          setting  $D_M = D_M \cup f$ ;

25        finding the first  $e_i \in \partial f$  such that  $-e_i \in \partial D_M$ ;

      replacing  $-e_i$  in  $\partial D_M$ , respectively, by  $\{e_{(i+1)}, e_{(i+2)}\}$ ;

      putting all the neighboring faces that share an edge with  $f$  in the mesh representation and that are not in  $D_M$  or  $Q$  into  
30  $Q$ ; and



removing all adjacent oriented edges in  $\partial D_M$  that are opposite each other in sign.

39. A method for remeshing a mesh representation,  $M$ , of a  
5 surface, the method comprising the steps of:

- a) subdividing the mesh,  $M$ , using loop subdivision;
- b) simplifying the mesh using edge collapse that uses a minimum edge length criteria.
- c) repeat steps a) and b) until all angles on the mesh  
10 representation,  $M$ , are acute; and
- d) outputting the remeshed representation,  $M$ , of the surface.

40. A method for converting a mesh representation,  $M$ , of a  
15 surface, the method comprising the steps of:

- receiving the mesh representation,  $M$ ;
- computing a conformal representation of the mesh representation,  $M$ ;
- computing the gradient of the mesh representation,  $M$ ;
- 20 computing the zero points of the mesh representation,  $M$ ;
- decomposing the mesh representation,  $M$ , into canonical patches using integration lines along the gradient and through the zero points;
- conformally mapping the canonical patches onto a respective  
25 rectangle;
- constructing a tensor product spline surface on the surface of the rectangle; and
- matching the control points on the boundary to make the parameterization globally smooth.

30

41. The method of claim 39 wherein the mesh representation,  $M$ , is discontinuous on the boundaries and wherein the parameterization is continuous on the boundaries.

5 42. A method for conformally mapping a mesh representation of a scanned medical image,  $M$ , of a body part to a canonical surface, the method comprising the steps of:

- a) computing the Gauss map, mapping  $M$  to  $S^2$ ;
- b) computing the Laplacian at each vertex  $u$  of  $M$ ,  $\Delta\phi(u)$ ;
- 10 c) projecting  $\Delta\phi(u)$  to the tangent space of  $\phi(u) \in S^2$ ;
- d) updating  $\phi(u)$  along the negative projected  $\Delta\phi(u)$ ;
- e) computing the center of mass of  $\phi(u)$ ,  $mc(\phi)$ ;
- f) shifting the center of mass to the center of  $S^2$ ;
- g) renormalizing  $\phi(u)$  to be on  $S^2$ ; and
- 15 h) repeating steps b)-g) for all vertices, until the projected Laplacian equals zero.

43. A method for animating a mesh representation,  $M$ , of a surface from a first mesh representation,  $M_1$ , to a second mesh  
20 representation,  $M_2$ , the method comprising the steps of:

- removing at least one feature point common to both  $M_1$  and  $M_2$ ;
- doubling each of  $M_1$  and  $M_2$ ;
- conformally mapping  $M_1$  and  $M_2$  to first and second canonical surfaces, forming first and second mapped surfaces;
- 25 computing a holomorphic 1-form for said first and second mapped surfaces;
- computing a cohomology basis of said first and second mapped surfaces;
- locating first and second zero points on said first and  
30 second mapped surfaces;

computing first and second gradients of said first and second mapped surfaces at said first and second zero points, respectively;

decomposing the first and second mesh representations,  $M_1$  and  
5  $M_2$ , into first and second sets of canonical patches using integration lines along the first and second gradients and through the first and second zero points, respectively;

conformally mapping the first and second sets of patches onto first and second rectangles, respectively;

10 matching said first and second mapped patches on said first and second rectangles to form a map therebetween;

selecting at least one control point on each of said first and second mapped patches on said first and second rectangles; and

15 using a BSpline to generate a smooth transition of said mapping.

44. A method of texture mapping on a mesh representation,  $M$ , representative of a surface, the method comprising the steps of:

20 removing at least one feature point on  $M$ ;

doubling  $M$ ;

conformally mapping  $M$  to a canonical surface, forming a mapped surface;

computing a holomorphic 1-form for said mapped surface;

25 computing a cohomology basis of said mapped surface;

locating zero points on said mapped surface;

computing a gradient of said mapped surface;

decomposing said mesh representation,  $M$ , into at least two canonical patches using integration lines along said gradient and  
30 through said zero point, respectively;

conformally mapping said at least two canonical patches onto at least two rectangles;

growing said at least two rectangles until respective  
boundaries between said at least two rectangles meet;

fixing the boundaries between said at least two rectangles;  
and

5 solving the Dirichlete problem for the uncovered regions of  
said at least two rectangles.

45. A method of volumetric harmonic mapping of a mesh  
representation,  $M$ , of a 3-dimensional manifold, the method  
10 comprising the steps of:

computing the harmonic energy of a mapping  $f : M \rightarrow R^3$ , where  
the harmonic energy is given by  $E(f) = \sum_{[u,v] \in M} k_{uv} \|f(u) - f(v)\|$

and  $k_{uv} = \frac{1}{48} \sum_{\theta} \cot(\theta)$ , where  $\theta$  is the dihedral angle opposite to the  
given edges;

15 minimizing the harmonic energy using the conjugate gradient  
method to obtain the harmonic mapping,  $f$ .

46. The method of claim 44 wherein the representation  $M$  of the  
3-dimensional manifold is a magnetic resonance image of a body  
20 area of interest, and wherein the mapping  $f : M \rightarrow R^3$  is a map onto  
a canonical sphere of the body area of interest.

47. The method of claim 45 further including using said map onto  
said canonical sphere of said body area of interest for surgical  
25 planning.

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